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LIII. *The Resolution of a General Proposition for Determining the horary Alteration of the Position of the Terrestrial Equator, from the Attraction of the Sun and Moon: With some Remarks on the Solutions given by other Authors to that difficult and important Problem. By Mr. Tho. Simpson, F. R. S*

Read Dec. 22,
1757.

SINCE the time, that that excellent Astronomer, my much honoured friend Dr. Bradley, published his observations and discoveries concerning the inequalities of the precession of the equinox, and of the obliquity of the ecliptic, depending on the position of the lunar nodes, mathematicians, in different parts of Europe, have set themselves diligently to compute, from physical principles, the effects produced by the sun and moon, in the position of the terrestrial equator; and to examine whether these effects do really correspond with the observations

Two papers on this subject have already appeared in the Philosophical Transactions; in which the authors have shewn evident marks of skill and penetration. There is, nevertheless, one part of the subject, that seems to have been passed over without a due degree of attention, as well by both those gentlemen, as by Sir Isaac Newton himself.

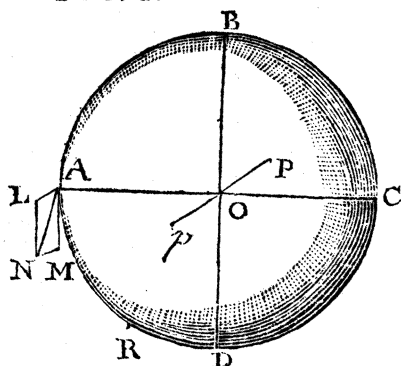
This part, which, upon account of physical difficulties, is indeed somewhat slippery and perplexing, I shall make the principal subject of this essay.

GENE-

GENERAL PROPOSITION.

Supposing an homogeneous sphere OABCD (Fig. 1.) revolving uniformly about its centre, to be acted on at the extremity A of the radius OA, in a direction AL perpendicular to the plane of the equator ABCD, and parallel to the axis of rotation Pp, by a given force, tending to generate a new motion of rotation at right angles to the former; It is proposed to determine the change, that will arise in the direction of the rotation in consequence of the said force.

FIG. 1.



Let F denote the given force, whereby the motion about the axis Pp is disturbed, supposing f to represent the centrifugal force of a small particle of matter in the circumference of the equator, arising from the sphere's rotation; and let the whole number of such particles, or the content of the sphere, be denoted by c : let also the momentum of rotation of the whole sphere, or of all the particles, be supposed, in proportion to the momentum of an equal number

of particles, revolving at the distance OA of the remotest point A , as n is to *unity*.

It is well known, that the centripetal force, whereby any body is made to revolve in the circumference of a circle, is such, as is sufficient to generate all the motion in the body, in a time equal to *that*, wherein the body describes an arch of the circumference, equal in length to the radius. Therefore, if we here take the arch $AR = OA$, and assume m to express the time, in which that arch would be uniformly described by the point A , the *motion* of a particle of matter at A (whose central force is represented by f) will be equal to *that*, which might be uniformly generated by the force f , in the time m ; and the motion of as many particles (revolving, all, at the same distance) as are expressed by cn (which, by hypothesis, is equal to the momentum of the whole body), will, consequently, be equal to the momentum, that might be generated by the force $f \times cn$, in the same time m . Whence it appears, that the momentum of the whole body about its axe Pp is in proportion to the momentum generated in a given particle of time m' , by the given force F in the direction AL , as $ncf \times m$ is to $F \times m'$, or, as *unity* to $\frac{F}{ncf} \times \frac{m'}{m}$ (because the quantities of motion produced by unequal forces, in unequal times, are in the ratio of the forces and of the times, conjunctly). Let, therefore, AL be taken in proportion to AM , as $\frac{F}{ncf} \times \frac{m'}{m}$ is to *unity* (supposing AM to be a tangent to the circle $ABCD$ in A), and let the parallelogram $AMNL$ be completed; drawing also the diagonal AN ; then, by
the

the composition of forces, the angle NAM (whose tangent, to the radius OA , is expressed by $OA \times \frac{F}{ncf} \times \frac{m'}{m}$) will be the change of the direction of the rotation, at the end of the aforesaid time (m'). But, this angle being exceeding small, the tangent may be taken to represent the measure of the angle itself; and, if Z be assumed to represent the arch described by A , in the same time (m') about the center O , we shall also have $\frac{m'}{m} = \frac{Z}{AR} = \frac{Z}{AO}$, and consequently $OA \times \frac{F}{ncf} \times \frac{m'}{m} = Z \times \frac{F}{ncf}$. From whence it appears, that the angle expressing the change of the direction of the rotation, during any small particle of time, will be in proportion to the angle described about the axe of rotation in the same time, as $\frac{F}{ncf}$ is to unity. *Q. E. I.*

Altho', in the preceding proposition, the body is supposed to be a perfect sphere, the solution, nevertheless, holds equally true in every other species of figures, as is manifest from the investigation. It is true, indeed, that the value of n will not be the same in these cases, even supposing those of c , f and F to remain unchanged; except in the spheroid only, where, as well as in the sphere, n will be $= \frac{2}{3}$; the momentum of any spheroid about its axis being 2-5ths of the momentum of an equal quantity of matter placed in the circumference of the equator, as is very easy to demonstrate.

But to shew now the use and application of the general proportion here derived, in determining the regrefs of the equinoctial points of the terrestrial
H h h 2
spheroid,

But it is demonstrated by Sir Isaac Newton, and by other authors, that the force of all the particles, or of all the matter in the whole spheroid $APap$, to turn *it* about its center, is equal to $\frac{1}{3}$ th of the force of a quantity of matter, placed at A , equal to the excess of the matter in the whole spheroid above *that* in the inscribed sphere, whose axis is Pp . Now this excess (assuming the ratio of π to 1, to express *that* of the area of a circle to the square of the radius) will be truly represented by $\frac{4\pi}{3} \times OP \times \overline{OA^2 - OP^2}$;

and, consequently, the force of all the matter in the whole earth, by $\frac{3tt}{TT} \times \frac{AK}{OA} \times \frac{OK}{OA} \times \frac{4\pi}{15} \times OP \times \overline{OA^2 - OP^2}$.

Let, therefore, this quantity be now substituted for F , in the general formula $\frac{F}{ncf}$, writing, at the same

time, $\frac{4\pi}{3} \times OA^2 \times OP$, and $\frac{3}{7}$, in the place of their equals c and n ; by which means we have (here)

$$\frac{F}{ncf} = \frac{3tt}{2TT} \times \frac{OA^2 - OP^2}{OA^2} \times \frac{AK \times OK}{OA^2}. \text{ Put the given}$$

quantity $\frac{3tt}{2TT} \times \frac{OA^2 - OP^2}{OA^2} = k$; and let the angle

$E Ae$ represent the horary alteration of the position of the terrestrial equator, arising from the force F (here determined), and let the arch Ee be the regress of the equinoctial point E , corresponding thereto: then, in the triangle $E Ae$ (considered as spherical) it will be $\sin. e : \sin. AE (:: \sin. EAe : \sin. Ee)$

$$:: EAe : Ee \left(= \frac{\sin. AE \times EAe}{\sin. E} \right) = k \times \frac{\sin. AE}{\sin. E} \times \frac{AK \times OK}{OA^2} = k \times \frac{\sin. AE \times \cos. AH \times \sin. AH}{\sin. E}. \text{ But in}$$

the triangle EHA , right-angled at A (where HA is supposed

supposed to represent the sun's declination, AE his right ascension, and HE his distance from the equinoctial point E*) we have (*per spherics*)

$$\text{fin. AE} : 1 \text{ (rad.)} :: \text{co-t. E} : \text{co-t. AH},$$

$$\overline{\text{fin. AH}}^2 : \overline{\text{fin. EH}}^2 :: \overline{\text{fin. E}}^2 : 1^2 \text{ (rad.}^2\text{)}$$

From whence we get, $\text{fin. AE} \times \text{co-t. AH} \times \overline{\text{fin. AH}}^2 = \overline{\text{fin. EH}}^2 \times \text{co-t. E} \times \overline{\text{fin. E}}^2$. But $\text{co-t. AH} \times \text{fin. AH} = \text{co-f. AH} \times 1 \text{ (rad.)}$, and $\text{co-t. E} \times \text{fin. E} = \text{co-f. E} \times 1 \text{ (rad.)}$: therefore $\text{fin. AE} \times \text{co-f. AH} \times \text{fin. AH} = \overline{\text{fin. EH}}^2 \times \text{co-f. E} \times \text{fin. E}$; and, consequently, $k \times \frac{\text{fin. AE} \times \text{co-f. AH} \times \text{fin. AH}}{\text{fin. E}} = k \times \text{co-f. E} \times \overline{\text{fin. EH}}^2 (= Ee)$.

Let, now, the sun's longitude EH be denoted by Z (considered as a flowing quantity); then, $\overline{\text{fin. Z}}^2$ being $= \frac{1}{2} - \frac{1}{2} \text{co-f. } 2Z$, we shall have $k \times \text{co-f. E} \times \overline{\text{fin. EH}}^2 = \frac{1}{2} k \times \text{co-f. E} \times \overline{1 - \text{co-f. } 2Z}$. But the angle described about the axe of rotation Pp, in the time that the sun's longitude is augmented by the particle \dot{Z} , will be $= \frac{T}{t} \times \dot{Z}$. Therefore (by the general proposition) we have, as $1 : \frac{1}{2} k \times \text{co-f. E} \times \overline{1 - \text{co-f. } 2Z} :: \frac{T}{t} \times \dot{Z} : \frac{1}{2} k \times \frac{T}{t} \times \text{co-f. E} \times \dot{Z} - \dot{Z} \text{co-f. } 2Z$, the true regrefs of the equinoctial point E, during

* No error arises from considering the triangles E Ae and AEH, as being formed on the surface of a sphere, tho' the earth itself is not accurately such. The angle (E A e) representing the effect of the solar force, is properly referred to the surface of a sphere; therefore (after the measure thereof is truly determined) the figure APap is itself taken as a sphere, in order to avoid the trouble of introducing a new scheme.

that

that time: whose fluent, $\frac{1}{2} k \times \frac{T}{t} \times \text{co-f. } E \times \overline{Z} = \frac{1}{2} \sin. 2 Z$, will consequently be the total regrefs of the point E, in the time that the fun, by his apparent motion, describes the arch HE or Z; which, on the fun's arrival at the solstice, becomes barely $= \frac{1}{2} k \times \frac{T}{t} \times \text{co-f. } E \times \text{an arch of } 90^\circ$: the quadruple whereof, or $\frac{1}{2} k \times \frac{T}{t} \times \text{co-f. } E \times 360^\circ \left(= \frac{3t}{4T} \times \frac{OA^2 - OP^2}{OA^2} \times \text{co-f. } E \times 360^\circ \right)$ is therefore the whole annual precession of the equinox caused by the sun. This, in numbers (taking $\frac{OP}{OA} = \frac{229}{230}$) comes out $\frac{3}{4 \times 366\frac{1}{2}} \times \frac{2}{230\frac{1}{2}} \times 0.917176 \times 360^\circ = 21'' 6'''$.

The very ingenious M. Silvabelle, in his essay on this subject, inserted in the 48th volume of the Philosophical Transactions, makes the quantity of the annual precession of the equinox, caused by the sun, to be the half, only, of what is here determined. But this gentleman appears to have fallen into a twofold mistake. First, in finding the *momenta of rotation* of the terrestrial spheroid, and of a very slender ring; at the equator thereof; which *momenta* he refers to an axis perpendicular to the plane of the sun's declination, instead of the proper axe of rotation, standing at right angles to the plane of the equator. The difference, indeed, arising from thence, with respect to the spheroid (by reason of its near approach to a sphere) will be inconsiderable; but, in the ring, the case will be quite otherwise; the equinoctial points thereof being made to recede just twice as fast

as they ought to do. This may seem the more strange, if regard be had to the conclusions, relating to the nodes of a satellite, derived from this very assumption. But, that these conclusions are true, is owing to a second, or subsequent mistake, at Art. 27; where the measure of the sun's force is taken the half, only, of the true value; by means whereof the motion of the equinoctial points of the ring is reduced to its proper quantity, and the motion of the equinoctial points of the terrestrial spheroid, to the half of what it ought to be.

That expert geometrician M. Cha. Walmisley, in his Essay on the Precession of the Equinox, printed in the last volume of the Philosophical Transactions, has judiciously avoided all mistakes of this last kind, respecting the sun's force, by pursuing the method, pointed out by Sir Isaac Newton; but, in determining the effect of that force, has fallen into others, not less considerable than those above adverted to.

In his third Lemma, the momentum of the whole Earth, about its diameter, is computed on a supposition, that the momentum or force of each particle is proportional to its distance from the axis of motion, or barely as the quantity of motion in such particle, considered abstractedly. No regard is, therefore, had to the lengths of the unequal levers, whereby the particles are supposed to receive and communicate their motion: which, without doubt, ought to have been included in the consideration.

In his first proposition, he determines, in a very ingenious and concise manner, the true annual motion of the nodes of a ring (or of a single satellite) at the earth's equator, revolving with the earth itself, about
its

its center, in the time of one fiderial day. This motion he finds to be $= \frac{3 \text{ co-f. } 23^{\circ} 29'}{4 \text{ rad.}} \times \frac{1}{366\frac{1}{4}} \times 360^{\circ}$. Then, in order to infer from thence, the motion of the equinoctial points of the earth itself, he, first, diminishes that quantity, in the ratio of 2 to 5: Because (as is demonstrated by Sir Isaac Newton in his 2d Lemma) the whole force of all the particles situated without the surface of a sphere, inscribed in the spheroid, to turn the body about its center, will be only 2-5ths of the force of an equal number of particles uniformly disposed round the whole circumference of the equator, in the fashion of a ring. The quantity $\left(\frac{3 \text{ co-f. } 23^{\circ} 29'}{4 \text{ rad.}} \times \frac{2}{5} \times \frac{1}{366\frac{1}{4}} \times 360^{\circ} \right)$ thus arising, will, therefore, express the true motion of the equinoctial points of a ring, equal in quantity of matter to the excess of the whole earth above the inscribed sphere, when the force whereby the ring tends to turn about its diameter is supposed equal to the force whereby the earth itself tends to turn about the same diameter, in consequence of the sun's attraction. Thus far our author agrees with Sir Isaac Newton; but, in deriving from hence the motion of the equinoctial points of the earth itself, he differs from him; and, in the corollary to his third Lemma, assigns the reasons, why he thinks Sir Isaac Newton, in this particular, has *wandered a little from the truth*. Instead of diminishing the quantity above exhibited (as Sir Isaac has done) in the ratio of all the motion in the ring to the motion in the whole earth, he diminishes it in the ratio of the motion of all the matter above the surface of the inscribed sphere to the motion of the whole earth: which matter, tho' equal

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to that of the ring, has nevertheless a different momentum, arising from the different situation of the particles in respect to the axis of motion.

But since the aforefaid quantity, from whence the motion of the earth's equinox is derived, as well by this gentleman, as by Sir Isaac Newton, expresses truly the annual regrefs of the equinoctial points of the ring (and not of the hollow figure formed by the faid matter, which is greater, in the ratio of 5 to 4) it seems, at least, as reasonable to suppose, that the faid quantity, to obtain from thence the true regrefs of the equinoctial points of the earth, ought to be diminished in the former of the two ratios above specified, as that it should be diminished in the latter. But, indeed, both these ways are defective, even supposing the momenta to have been truly computed; the ratio, that ought to be used here, being that of the momenta of the ring and earth about the proper axe of rotation of the two figures, standing at right-angles to the plane of the ring and of the equator. Now this ratio, by a very easy computation, is found to be as $\overline{230}^2 - \overline{229}^2$ to $\frac{2}{3}$ of $\overline{230}^2$; whence the quantity sought comes out = $\frac{3 \text{ co-f. } 23^\circ 29'}{4 \text{ rad.}}$

$\times \frac{1}{366\frac{1}{4}} \times \frac{\overline{230}^2 - \overline{229}^2}{230^2} \times 360^\circ = 21'' 6'''$: which is the same that we before found it to be, and the double of what this author makes it.

What has been said hitherto, relates to that part of the motion only, arising from the force of the sun. It will be but justice to observe here, that the effect of the moon, and the inequalities depending on the position of her nodes, are truly assigned by both the gentlemen

gentlemen above-named; the ratio of the diameters of the earth, and the density of the moon being so assumed, as to give the maxima of those inequalities, such as the observations require: in consequence whereof, and from the law of the increase and decrease (which is rightly determined by theory, tho' the absolute quantity is not) a true solution, in every other circumstance, is obtained.

The freedom, with which I have expressed myself, and the liberty I have here taken, to animadvert on the works of men, who, in many places, have given incontestible proofs of skill and genius, may, I fear, stand in need of some apology. 'Tis possible I may be thought too peremptory. Indeed, I might have delivered my sentiments with more caution and address: but, had not I imagined myself quite clear in what has been advanced, from a multitude of concurrent reasons, I should have thought it too great a presumption to have said any thing at all here, on this subject. The great regard I have for this Society, of which I have the honour to be a member, will, I hope, be considered as the motive for my having attempted to rectify some oversights, that have occurred in the works of this learned body.